

Weak decays of medium and heavy Λ -hypernuclei

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Abstract

We have made a new evaluation of the Λ decay width in nuclear matter within the Propagator Method. Through the Local Density Approximation it is possible to obtain results in finite nuclei. We have also studied the dependence of the widths on the NN and ΛN short range correlations. Using reasonable values for the parameters that control these correlations, as well as realistic nuclear densities and Λ wave functions, we reproduce, for the first time, the experimental non-mesonic widths in a wide range of mass numbers (from medium to heavy hypernuclei).

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I. INTRODUCTION

A hypernucleus is a bound system made of by neutrons, protons and one or more hyperons. Among these *strange nuclei*, those which contain one Λ hyperon are the most stable with respect to the strong interaction and they are the subject of this paper. The study of hypernuclear physics may help in understanding some present problems related, for instance, to some aspects of weak interactions in nuclei, or to the origin of the spin-orbit interaction in nuclei. Besides, it is a good instrument to study the role of quark degrees of freedom in the hadron-hadron interactions at short distances and the renormalization properties of pions in the nuclear medium.

Nowadays we know some important features of the YN interaction [1]. For example, at intermediate distances the strong ΛN interaction is weaker than the NN one, and its spin-orbit term is very small. Moreover, the former has a smaller range than the NN one. From the study on mesonic decays of light hypernuclei we have evidence for strongly repulsive cores in the ΛN interaction at short distances [2], which automatically appears in quark based models [3,4]. These characteristics of the ΛN interaction are important, as we will see, for the evaluation of the decay rates of Λ -hypernuclei.

The most interesting hypernuclear decays are those involving weak processes, which directly concern the hyperon. The weak decay of hypernuclei occurs via two channels: the so called mesonic channel ($\Lambda \rightarrow \pi N$) and the non-mesonic one, in which the pion emitted from the weak hadronic vertex is absorbed by one or more nucleons in the medium ($\Lambda N \rightarrow NN$, $\Lambda NN \rightarrow NNN$, etc.). Obviously, the non-mesonic processes can also be mediated by the exchange of more massive mesons than the pion. The non-mesonic decay is only possible in nuclei and, nowadays, the study of the hypernuclear decay is the only practical way to get information on the weak process $\Lambda N \rightarrow NN$, especially on its parity conserving part. In fact, there are not experimental observations for this interaction using lambda beams. It is, however, under study the inverse reaction $pn \rightarrow \Lambda p$ at COSY [5] and RCNP [6].

The free Λ decay is compatible with the $\Delta I = 1/2$ isospin rule, which is also valid for the decay of other hyperons and for kaons (namely in non-leptonic strangeness changing processes). This rule is based on the experimental observation that the $\Lambda \rightarrow \pi^- p$ decay rate is twice the $\Lambda \rightarrow \pi^0 n$ one, but it is not yet understood on theoretical grounds. From theoretical calculations like the one in ref. [7] and from experimental measurements [8] there is some evidence that the $\Delta I = 1/2$ rule is broken in nuclear mesonic decay. However, this is essentially due to shell effects and might not be directly related to the weak process. A recent estimate of $\Delta I = 3/2$ contributions to the $\Lambda N \rightarrow NN$ reaction [9] found moderate effects on the hypernuclear decay rates. In the present calculation of the decay rates in nuclei we will assume this rule as valid. The momentum of the final nucleon in $\Lambda \rightarrow \pi N$ is about 100 MeV for Λ at rest, so this process is suppressed by the Pauli principle in nuclei (particularly in heavy systems). It is strictly forbidden in infinite nuclear matter (where $k_F^0 \simeq 270$ MeV), but in finite nuclei it can occur because of three important effects: 1) in nuclei the hyperon has a momentum distribution that allows larger momenta for the final

nucleon, 2) the final pion feels an attraction by the medium such that for fixed momentum it has a smaller energy than the free one and consequently, due to energy conservation, the final nucleon again has more chance to come out above the Fermi surface, and 3) on the nuclear surface the local Fermi momentum is smaller than k_F^0 and favours the decay. Nevertheless, the mesonic width decreases fastly as the mass number A of the hypernucleus increases [7]. From the study of the mesonic channel it could be possible to extract important information on the pion-nucleus optical potential, which we do not know today in a complete form. In fact, the mesonic rate is very sensitive to the pion self-energy in the medium [7].

The final nucleons in the non-mesonic process $\Lambda N \rightarrow NN$ emerge with large momenta ($\simeq 420$ MeV), so this decay is not forbidden by the Pauli principle. On the contrary, apart from very light hypernuclei (the s -shell ones), it dominates over the mesonic decay. The non-mesonic channel is characterized by large momentum transfers, so that the details of the nuclear structure do not have a substantial influence while the NN and ΛN Short Range Correlations (SRC) turn out to be very important. There is an anticorrelation between mesonic and non-mesonic decay modes such that the total lifetime is quite stable from light to heavy hypernuclei [8,10]: $\tau_{exp} = (0.5 \div 1) \tau_{free}$.

Nowadays, the main problem concerning the weak decay rates is to reproduce the experimental values for the ratio Γ_n/Γ_p between the neutron and the proton induced widths $\Lambda n \rightarrow nn$ and $\Lambda p \rightarrow np$. The theoretical calculations underestimate the experimental data for all the considered hypernuclei [8,9,11–14]:

$$\left\{ \frac{\Gamma_n}{\Gamma_p} \right\}^{Th} \ll \left\{ \frac{\Gamma_n}{\Gamma_p} \right\}^{Exp} \quad 0.5 \lesssim \left\{ \frac{\Gamma_n}{\Gamma_p} \right\}^{Exp} \lesssim 2. \quad (1.1)$$

In the One Pion Exchange (OPE) approximation the values for this ratio are $0.1 \div 0.2$. On the other hand the OPE model has been able to reproduce the 1-body stimulated non-mesonic rates $\Gamma_1 = \Gamma_n + \Gamma_p$ for light and medium hypernuclei [11,12,14]. In order to solve this problem many attempts have been made up to now, but without success. Among these we recall the inclusion in the $\Lambda N \rightarrow NN$ transition potential of mesons heavier than the pion [11,13,14], the inclusion of interaction terms that violate the $\Delta I = 1/2$ rule [9] and the description of the short range baryon-baryon interaction in terms of quark degrees of freedom [12]. This last calculation is the only one that has found a consistent (but not sufficient) increase of the neutron to proton ratio with respect to the OPE one. However, this calculation is only made for s -shell hypernuclei and their effective quark-lagrangian does not reproduce the experimental ratio between the $\Delta I = 1/2$ and $\Delta I = 3/2$ transition amplitudes for the Λ free decay.

The analysis of the ratio Γ_n/Γ_p is influenced by the 2-nucleon induced process $\Lambda NN \rightarrow NNN$. By assuming that the meson produced in the weak vertex is mainly absorbed by a neutron-proton strongly correlated pair, the 3-body process turns out to be $\Lambda np \rightarrow nnp$, so that a considerable fraction of the measured neutrons could come from this channel and not only from the $\Lambda n \rightarrow nn$ and $\Lambda p \rightarrow np$ ones. In this way it might be possible to explain the large experimental Γ_n/Γ_p ratios, which originally have been analyzed without taking

into account the 2-body stimulated process. Nevertheless, the situation is far from being clear and simple. The new non-mesonic mode was introduced in ref. [15] and its calculation was improved in ref. [16], where the authors found that the inclusion of the new channel would lead to larger values of the Γ_n/Γ_p ratios extracted from the experiment, somehow more in disagreement with theoretical estimates. However, in the hypothesis that only two nucleons from the 3-body decay are detected, the reanalysis of the experimental data would lead to smaller ratios [8]. These observations show that Γ_n/Γ_p is sensitive to the energy spectra of the emitted nucleons, whose calculation also requires a careful treatment of the nucleon Final State Interaction. In ref. [17] the energy distributions were calculated using a Monte Carlo simulation to describe the final state interactions. A direct comparison of those spectra with the experimental ones favours Γ_n/Γ_p values around $2 \div 3$ (or higher), in disagreement with the OPE predictions. However, it was also pointed out the convenience of measuring the number of protons per decay event. This observable, which can be measured from delayed fission events in the decay of heavy hypernuclei, gives a more reliable ratio Γ_n/Γ_p and is less sensitive to details of the Monte Carlo simulation determining the final shape of the spectra.

In this paper we present a new evaluation of the decay rates for medium to heavy hypernuclei based on the Propagator Method of ref. [18], which allows a unified treatment of all the decay channels. The parameters of the model are adjusted to reproduce the non-mesonic width of ${}_{\Lambda}^{12}\text{C}$ and the decay rates of heavier hypernuclei are predicted. We also discuss how the new model affects the energy spectrum of the emitted nucleons, in the hope of obtaining a ratio Γ_n/Γ_p more in agreement with the experimental observation.

The paper is organized as follows. In Sec. II we present the model used for the calculation of the decay rates. Our results are presented and discussed in Sec. III. We first study the sensitivity of the decay rates to the parameters defining the NN and ΛN short range correlations as well as to the nuclear density and Λ wave functions. We then obtain the widths for various hypernuclei and discuss the energy distribution of the nucleons from the weak decays. Our conclusions are given in Sec. IV.

II. PROPAGATOR METHOD

The Λ decay in nuclear systems can be studied in Random Phase Approximation (RPA) using the Propagator Method [16,18]. This technique provides a unified picture of the different decay channels and it is equivalent to the standard Wave Function Method (WFM) [19], used by other authors in refs. [7,9,11,13,14]. The calculation of the widths is usually performed in nuclear matter, and then extended to finite nuclei via the Local Density Approximation (LDA). For the calculation of the mesonic rates the WFM is more reliable than the propagator method in LDA since this channel is rather sensitive to the shell structure of the hypernucleus, given the small energies involved. Moreover, it is advisable to avoid the use of the LDA for very light systems and we will make the calculation starting from ${}_{\Lambda}^{12}\text{C}$.

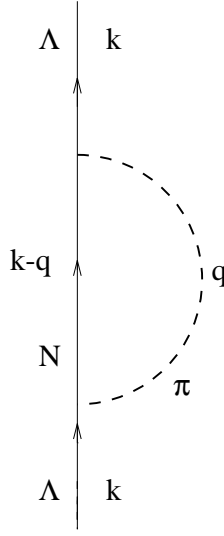


FIG. 1. Λ self energy in nuclear matter

On the other hand, the propagator method in LDA offers the possibility of calculations over a broad range of mass numbers.

The method was introduced in ref. [18] and we briefly summarize it here for clarity. The $\Lambda \rightarrow \pi N$ effective lagrangian is:

$$\mathcal{L}_{\Lambda\pi N} = Gm_\pi^2 \bar{\psi}_N (A + B\gamma_5) \boldsymbol{\tau} \cdot \boldsymbol{\phi}_\pi \psi_\Lambda + h.c., \quad (2.1)$$

where the values of the weak coupling constants $G \simeq 2.211 \cdot 10^{-7}/m_\pi^2$, $A = 1.06$, $B = -7.10$ are fixed on the free Λ decay. The constants A and B determine the strengths of the parity violating and parity conserving $\Lambda \rightarrow \pi N$ amplitudes respectively. In order to enforce the $\Delta I = 1/2$ rule, in eq. (2.1) the hyperon is assumed to be an isospin spurion with $I_z = -1/2$. To calculate the Λ width in nuclear matter we start with the imaginary part of the Λ self-energy:

$$\Gamma_\Lambda = -2\text{Im}\Sigma_\Lambda. \quad (2.2)$$

By the use of Feynman rules, from fig. 1 it is easy to obtain the Λ self-energy in the following form:

$$\Sigma_\Lambda(k) = 3i(Gm_\pi^2)^2 \int \frac{d^4q}{(2\pi)^4} \left\{ S^2 + \frac{P^2}{m_\pi^2} \mathbf{q}^2 \right\} F_\pi^2(q) G_N(k-q) G_\pi(q). \quad (2.3)$$

Here, $S = A$, $P = m_\pi B/2m_N$, while the nucleon and pion propagators in nuclear matter are respectively:

$$G_N(p) = \frac{\theta(|\mathbf{p}| - k_F)}{p_0 - E_N(\mathbf{p}) - V_N + i\epsilon} + \frac{\theta(k_F - |\mathbf{p}|)}{p_0 - E_N(\mathbf{p}) - V_N - i\epsilon}, \quad (2.4)$$

and:

$$G_\pi(q) = \frac{1}{q_0^2 - \mathbf{q}^2 - m_\pi^2 - \Sigma_\pi^*(q)}. \quad (2.5)$$

In the above, $p = (p_0, \mathbf{p})$ and $q = (q_0, \mathbf{q})$ denote four-vectors, k_F is the Fermi momentum, E_N is the nucleon total free energy, V_N is the nucleon binding energy, and Σ_π^* is the pion proper self-energy in nuclear matter. Moreover, in eq. (2.3) we have included a monopole form factor for the $\pi\Lambda N$ vertex:

$$F_\pi(q) = \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - q_0^2 + \mathbf{q}^2} \quad (2.6)$$

(the same is used for the πNN strong vertex), with cut-off $\Lambda_\pi = 1.2$ GeV. In fig. 2 we show the lowest order Feynman graphs for the Λ self-energy in nuclear matter. Diagram (a) represents the bare self-energy term, including the effects of Pauli principle and of binding on the intermediate nucleon. In (b) and (c) the pion couples to a particle-hole (p - h) and a Δ - h pairs, respectively. Diagram (d) is an insertion of s -wave pion self-energy at lowest order. In diagram (e) we show a $2p$ - $2h$ excitation coupled to the pion through s -wave πN interactions. Other $2p$ - $2h$ excitations, coupled in p -wave, are shown in (f), while (g) is a RPA iteration of diagram (b). It is possible to evaluate the integral over q_0 in (2.3), and the Λ self-energy (eq. (2.2)) in nuclear matter becomes [18]:

$$\begin{aligned} \Gamma_\Lambda(\mathbf{k}, \rho) = & -6(Gm_\pi^2)^2 \int \frac{d\mathbf{q}}{(2\pi)^3} \theta(|\mathbf{k} - \mathbf{q}| - k_F) \theta(k_0 - E_N(\mathbf{k} - \mathbf{q}) - V_N) \\ & \times \text{Im}\alpha(q) \big|_{q_0=k_0-E_N(\mathbf{k}-\mathbf{q})-V_N}, \end{aligned} \quad (2.7)$$

where

$$\begin{aligned} \alpha(q) = & \left\{ S^2 + \frac{P^2}{m_\pi^2} \mathbf{q}^2 \right\} F_\pi^2(q) G_\pi^0(q) + \frac{\tilde{S}^2(q) U(q)}{1 - V_L(q) U(q)} \\ & + \frac{\tilde{P}_L^2(q) U(q)}{1 - V_L(q) U(q)} + \frac{\tilde{P}_T^2(q) U(q)}{1 - V_T(q) U(q)}. \end{aligned} \quad (2.8)$$

In eq. (2.7) the first θ function forbids intermediate nucleon momenta (see fig. 1) smaller than the Fermi momentum and the second one requires the pion energy q_0 to be positive. Moreover, the Λ energy, $k_0 = E_\Lambda(\mathbf{k}) + V_\Lambda$, contains a binding term. The pion lines of fig. 2 have been replaced in eq. (2.8) by the interactions \tilde{S} , \tilde{P}_L , \tilde{P}_T , V_L , V_T , which include π and ρ exchange modulated by the effect of short range correlations and whose expressions are given in the Appendix. The functions V_L and V_T represent the (strong) p - h interaction, including a short range Landau parameter g' , while \tilde{S} , \tilde{P}_L and \tilde{P}_T correspond to the lines connecting weak and strong hadronic vertices and contain another short range Landau parameter g'_Λ . Furthermore, in eq. (2.8):

$$G_\pi^0(q) = \frac{1}{q_0^2 - \mathbf{q}^2 - m_\pi^2}, \quad (2.9)$$

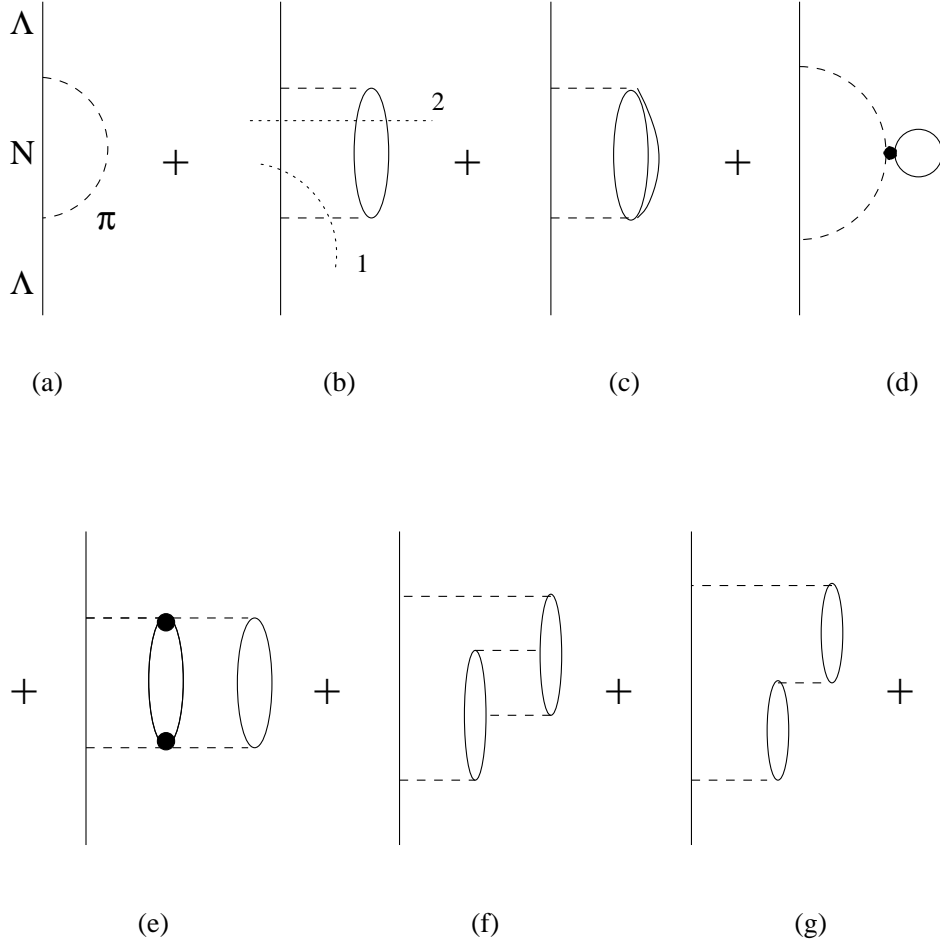


FIG. 2. Lowest order terms for the Λ self-energy in nuclear matter. The meaning of the various diagrams is explained in the text.

is the free pion propagator, while $U(q)$ contains the Lindhard functions for p - h and Δ - h excitations [20] and also accounts for $2p$ - $2h$ excitations:

$$U(q) = U_{ph}(q) + U_{\Delta h}(q) + U_{2p2h}(q). \quad (2.10)$$

It appears in eq. (2.8) within the standard RPA expression. Eq. (2.7) depends explicitly and through $U(q)$ on the nuclear matter density $\rho = 2k_F^3/3\pi^2$. The Lindhard functions U_{ph} , $U_{\Delta h}$ are normalized as in ref. [21] and U_{2p2h} is evaluated as in [16], that is, by calculating the available phase space for $2p$ - $2h$ excitations and by taking into account the experimental data on pionic atoms. $U(q)$ is related to the pion proper self-energy through:

$$\Sigma_\pi^*(q) = \Sigma_\pi^{(p)*}(q) + \Sigma_\pi^{(s)*}(q), \quad \Sigma_\pi^{(p)*}(q) = \frac{\frac{f_\pi^2}{m_\pi^2} \mathbf{q}^2 F_\pi^2(q) U(q)}{1 - \frac{f_\pi^2}{m_\pi^2} g_L(q) U(q)}, \quad (2.11)$$

where the Landau function $g_L(q)$ is given in the Appendix [see eq. (A12)], and $\Sigma_\pi^{(s)*}$ is the s -wave part of the self-energy. We will use the parametrization of ref. [22]: $\Sigma_\pi^{(s)*}(q) = -4\pi(1 + m_\pi/m_N)b_0\rho$, with $b_0 = -0.0285/m_\pi$. The function $\Sigma_\pi^{(s)*}$ is real (constant and positive), therefore it contributes only to the mesonic decay (diagram (d) in fig. 2 is the relative lowest order). On the contrary, the p -wave self-energy $\Sigma_\pi^{(p)*}$ is complex and attractive (that is, $\text{Re}\Sigma_\pi^{(p)*}(q) < 0$).

The decay widths in finite nuclei are obtained in LDA. In this approximation, the Fermi momentum becomes r -dependent (that is, a local Fermi sea of nucleons is introduced) and related again to the nuclear density by:

$$k_F(\mathbf{r}) = \left\{ \frac{3}{2}\pi^2 \rho(\mathbf{r}) \right\}^{1/3}. \quad (2.12)$$

Besides, the nucleon binding potential V_N also becomes r -dependent in LDA. In Thomas-Fermi approximation we assume:

$$\epsilon_F(\mathbf{r}) + V_N(\mathbf{r}) \equiv \frac{k_F^2(\mathbf{r})}{2m_N} + V_N(\mathbf{r}) = 0. \quad (2.13)$$

For the Λ binding energy we use instead the experimental value [23,24]. With these prescriptions we can then evaluate the decay width in finite nuclei through the relation:

$$\Gamma_\Lambda(\mathbf{k}) = \int d\mathbf{r} \, |\psi_\Lambda(\mathbf{r})|^2 \Gamma_\Lambda[\mathbf{k}, \rho(\mathbf{r})], \quad (2.14)$$

where ψ_Λ is the Λ wave function and $\Gamma_\Lambda[\mathbf{k}, \rho(\mathbf{r})]$ is given by eqs. (2.7), (2.8). This decay rate is valid for fixed Λ momentum \mathbf{k} . A further average over the Λ momentum distribution gives the total width:

$$\Gamma_\Lambda = \int d\mathbf{k} \, |\tilde{\psi}_\Lambda(\mathbf{k})|^2 \Gamma_\Lambda(\mathbf{k}), \quad (2.15)$$

which can be compared with experimental results.

The propagator method provides a unified picture of the decay widths. The imaginary part of a self-energy diagram requires placing simultaneously on-shell the particles of the considered intermediate state. For instance, diagram (b) in fig. 2 has two sources of imaginary part. One comes from cut 1, where the nucleon and the pion are placed on-shell. This term contributes to the mesonic channel: the final pion interacts with the medium through a p - h excitation and then escapes from the nucleus. Diagram (b) and further iterations lead to a renormalization of the pion in the medium which increases the mesonic rate by about two orders of magnitude in heavy nuclei [18]. The cut 2 in fig. 2(b) place a nucleon and a p - h pair on shell, so it is the lowest order contribution to the physical process $\Lambda N \rightarrow NN$.

The mesonic width Γ_M is calculated from

$$\alpha(q) = \left\{ S^2 + \frac{P^2}{m_\pi^2} \mathbf{q}^2 \right\} G_\pi(q), \quad (2.16)$$

by omitting $\text{Im}\Sigma_\pi^*$ in G_π , namely by replacing:

$$\text{Im}G_\pi(q) \rightarrow -\pi\delta(q_0^2 - \mathbf{q}^2 - m_\pi^2 - \text{Re}\Sigma_\pi^*(q)). \quad (2.17)$$

The one-body induced non-mesonic decay rate Γ_1 is obtained by substituting in eqs. (2.7), (2.8):

$$\text{Im} \frac{U(q)}{1 - V_{L,T}(q)U(q)} \rightarrow \frac{\text{Im}U_{ph}(q)}{|1 - V_{L,T}(q)U(q)|^2}, \quad (2.18)$$

that is by omitting the imaginary parts of $U_{\Delta h}$ and U_{2p2h} in the numerator. Indeed $\text{Im}U_{\Delta h}$ accounts for the $\Delta \rightarrow \pi N$ decay width, thus representing a contribution to the mesonic decay. There is no overlap between $\text{Im}U_{ph}(q)$ and the pole $q_0 = \omega(\mathbf{q})$ in eq. (2.17), so the separation of the mesonic and 2-body non-mesonic channels is unambiguous. The renormalized pion pole in eq. (2.16) is given by the dispersion relation:

$$\omega^2(\mathbf{q}) - \mathbf{q}^2 - m_\pi^2 - \text{Re}\Sigma_\pi^*[\omega(\mathbf{q}), \mathbf{q}] = 0, \quad (2.19)$$

with the constraint:

$$q_0 = k_0 - E_N(\mathbf{k} - \mathbf{q}) - V_N. \quad (2.20)$$

At the pion pole $\text{Im}U_{2p2h} \neq 0$, thus the 2-body induced non-mesonic width Γ_2 cannot be calculated using the prescription (2.18) with U_{2p2h} instead of U_{ph} in the numerator of the r. h. s. Part of the decay rate calculated in this way it is due to excitations of the renormalized pion and contributes to Γ_M . The 3-body non-mesonic rate is then calculated by subtracting Γ_M and Γ_1 from the total rate Γ_{TOT} , which we get via the full expression for α [Eq. (2.8)].

III. RESULTS AND DISCUSSION

Let us now discuss the numerical results one can obtain from the above illustrated formalism. We shall first study the influence of short range correlations and the Λ wave function on the decay width of ${}^{12}_{\Lambda}\text{C}$, which will be used as a testing ground for the theoretical framework in order to fix the parameters of our model. We will then obtain the decay widths of heavier hypernuclei and we will explore whether the refined model influences the energy distribution of the emitted particles, following the Monte Carlo procedure of ref. [17].

In order to evaluate the width (2.14) in LDA one needs to specify the nuclear density and the wave function for the Λ . The former is assumed to be a Fermi distribution:

$$\rho_A(r) = \frac{\rho_0(A)}{1 + e^{[r-R(A)]/a}} \quad \left[\rho_0(A) = \frac{A}{\frac{4}{3}\pi R^3(A) \{1 + [\frac{\pi a}{R(A)}]^2\}} \right], \quad (3.1)$$

with radius $R(A) = 1.12A^{1/3} - 0.86A^{-1/3}$ fm and thickness $a = 0.52$ fm. The Λ wave function is obtained from a Wood-Saxon (W-S) well which exactly reproduces the first two single particle eigenvalues (s and p levels) measured in Λ -hypernuclei.

A. Short range correlations and Λ wave function

A crucial ingredient in the calculation of the decay widths is the short range part of the NN and ΛN interactions. They are expressed by the functions $g_{L,T}(q)$ and $g_{L,T}^{\Lambda}(q)$, which are reported in the Appendix and contain the Landau parameters g' and g'_{Λ} . No experimental information is available on g'_{Λ} , while many constraints have been set on g' , for example by the well known quenching of the Gamow-Teller resonance. Realistic values of g' , within the framework of the ring approximation, are in the range $0.6 \div 0.7$ [25]. However, in the present context g' correlates not only p - h pairs but also p - h with $2p$ - $2h$ states. In order to fix these parameters we shall compare our calculations with the experimental non-mesonic width of ${}^{12}_{\Lambda}\text{C}$.

In fig. 3 we see how the total non-mesonic width for carbon depends on the Landau parameters. The rate decreases as g' increases. This characteristic is well established in RPA. Moreover, fixing g' , there is a minimum for $g'_{\Lambda} \simeq 0.4$ (almost independent of the value of g'). This is due to the fact that for $g'_{\Lambda} \ll 0.4$ the longitudinal p -wave contribution in eq. (2.8) dominates over the transverse one and the opposite occurs for $g'_{\Lambda} \gg 0.4$. We also remind that the s -wave interactions are independent of g'_{Λ} [eq. (A10)]. Moreover, the longitudinal p -wave $\Lambda N \rightarrow NN$ interaction [eq. (A8)] contains the pion exchange plus SRC, while the transverse p -wave $\Lambda N \rightarrow NN$ interaction [eq. (A9)] only contains repulsive correlations, so with increasing g'_{Λ} the p -wave longitudinal contribution to the width decreases, while the p -wave transverse part increases. From fig. 3 we see that there is a broad range of choices of g' and g'_{Λ} values which fit the experimental band. The latter represents the non-mesonic decay width which is compatible with both the BNL [26] and KEK [27] experiments. One should

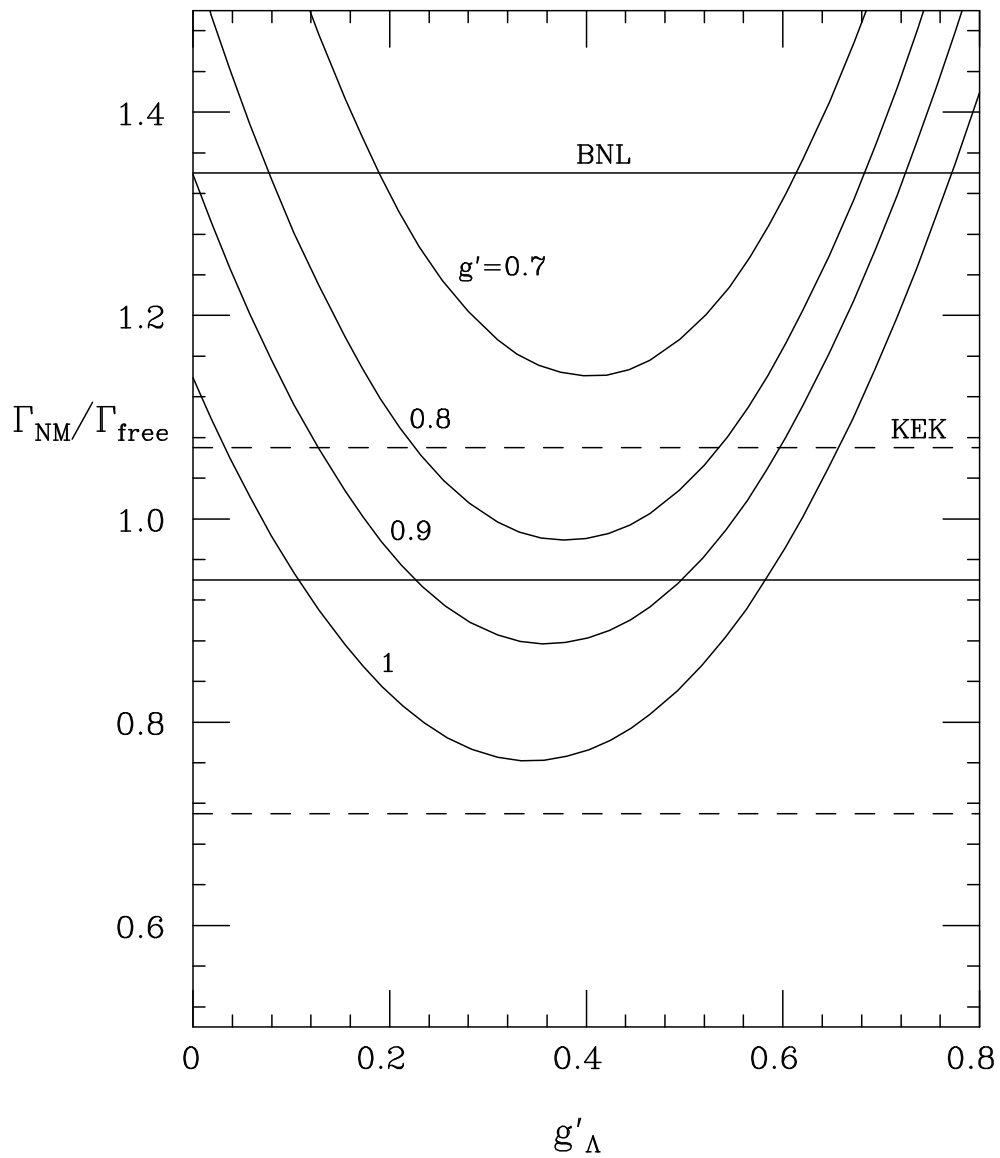


FIG. 3. Dependence of the non-mesonic width on the Landau parameters g' and g'_Λ for $^{12}_\Lambda\text{C}$. The experimental value from BNL [26] (KEK [27]) lies in between the horizontal solid (dashed) lines.

TABLE I. Wave function sensitivity

$^{12}_\Lambda\text{C}$	H.O.	Dover	New W-S	Microscopic	BNL [26]	KEK [27]	KEK New [30,31]
Γ_M	0.26	0.25	0.25	0.25	0.11 ± 0.27	0.36 ± 0.15	> 0.11
Γ_1	0.78	0.77	0.82	0.69			
Γ_2	0.15	0.15	0.16	0.13			
Γ_{NM}	0.93	0.92	0.98	0.81	1.14 ± 0.20	0.89 ± 0.18	< 1.03
Γ_{TOT}	1.19	1.17	1.23	1.06	1.25 ± 0.18	1.25 ± 0.18	1.14 ± 0.08

notice that the theoretical curves reported in fig. 3 contain the contribution of the 3-body process; should the latter be neglected (ring approximation) then one could get equivalent results with g' values smaller than the ones reported in the figure (typically $\Delta g' \simeq -0.1$). The phenomenology of the (e, e') quasi-elastic scattering suggests, in ring approximation, g' values of the order of 0.7. Here, by taking into account also $2p - 2h$ contributions, we shall use the “equivalent” value $g' = 0.8$, together with $g'_\Lambda = 0.4$.

We note that the values used in ref. [16], namely $g' = 0.615$ and $g'_\Lambda = 0.62$, would yield $\Gamma_1 = 1.26$ and $\Gamma_2 = 0.25$, adding to a non-mesonic width $\Gamma_{NM} = 1.51$, which is 50% larger than the experimental one. Thus the analysis performed here shows that the present data for $^{12}_\Lambda\text{C}$ favour a somewhat different but still reasonable g' value.

We shall now illustrate the sensitivity of our calculation to the Λ wave function in $^{12}_\Lambda\text{C}$. In addition to the W-S that reproduces the s and p levels, we also use a harmonic oscillator wave function with an “empirical” frequency ω [23,24], again obtained from the $s - p$ energy shift, the W-S wave function of Dover *et al.* [28] and the microscopic wave function calculated from a non-local self-energy using a realistic YN interaction in ref. [29]. The results are shown in table I, where they are compared with the experimental data from BNL [26] and KEK [27,30,31].

By construction, the chosen g' and g'_Λ reproduce the experimental decay widths using the W-S wave function which gives the right s and p levels in $^{12}_\Lambda\text{C}$. We note that it is possible to generate the microscopic wave function of ref. [29] for carbon via a local hyperon-nucleus W-S potential with radius 2.92 fm and depth -23 MeV. Although this potential reproduces fairly well the experimental s -level for the Λ in $^{12}_\Lambda\text{C}$, it does not reproduce the p -level. In this work we prefer to use a completely phenomenological Λ -nucleus potential that can easily be extended to heavier nuclei and reproduces the experimental Λ single particle levels as well as possible. Except for s -shell hypernuclei, where experimental data require Λ -nucleus potentials with a repulsive core at short distances [2], the Λ binding energies have been well reproduced by W-S potentials. We thus use a W-S potential with fixed diffuseness ($a = 0.6$ fm) and adjust the radius and depth to reproduce the s and p Λ -levels. The parameters of the potential for carbon are $R = 2.27$ fm and $V_0 = -32$ MeV.

To analyse the results of table I, we note that the microscopic wave function is substan-

TABLE II. W-S parameters

Λ^{A+1}_Z	R (fm)	V_0 (MeV)
$^{12}_\Lambda\text{C}$	2.27	-32.0
$^{28}_\Lambda\text{Si}$	3.33	-29.5
$^{40}_\Lambda\text{Ca}$	4.07	-28.0
$^{56}_\Lambda\text{Fe}$	4.21	-29.0
$^{89}_\Lambda\text{Y}$	5.07	-28.5
$^{139}_\Lambda\text{La}$	6.81	-27.5
$^{208}_\Lambda\text{Pb}$	5.65	-32.0

tially more extended than all the other wave functions used in the present study. The Dover parameters [28], namely $R = 2.71$ fm, $V_0 = -28$ MeV, give rise to a Λ wave function that is somewhat more extended than the new W-S one but is very similar to that obtained from a harmonic oscillator with a frequency of 10.9 MeV, adjusted to the $s - p$ energy shift in carbon. Consequently, the non-mesonic width from the Dover's wave function is very similar to the one obtained from the harmonic oscillator and slightly smaller than the new W-S one. The microscopic wave-function predicts the smallest non-mesonic widths due to the more extended Λ wave-function, which explores regions of lower density and thus has a smaller probability of interacting with one or more nucleons. From table I we also see that, against intuition, the mesonic width is quite insensitive to the Λ wave function. On this point we remind that for fixed Λ -momentum, the more extended is the wave function in r -space, the larger is the mesonic width, since the Pauli blocking effects on the emitted nucleon are reduced. But, when we make the integral over the Λ -momenta in LDA (eq. (2.15)), to more extended wave functions in r -space correspond less extended momentum distributions which tend to decrease the mesonic width. The two effects tend to cancel each other and Γ_M is insensitive to the different wave functions used in the calculation. In summary, different (but realistic) Λ wave functions give rise to total decay widths which may differ at most by 15%.

B. Decay widths of medium-heavy hypernuclei

Using the new W-S wave functions and the Landau parameters $g' = 0.8$, $g'_\Lambda = 0.4$ we have extended the calculation to heavier hypernuclei. We note that, in order to reproduce the experimental s and p levels for the hyperon we must use potentials with nearly constant depth, around $28 \div 32$ MeV, from medium to heavy hypernuclei (radii and depths of the used W-S potentials are quoted in table II).

Our results are shown in table III. We observe that the mesonic rate rapidly vanishes by increasing the mass number A . This is well known and it is related to the decreasing

TABLE III. Decay rates

$\Lambda^{A+1}Z$	Γ_M	Γ_1	Γ_2	Γ_{TOT}
$\Lambda^{12}\text{C}$	0.25	0.82	0.16	1.23
$\Lambda^{28}\text{Si}$	0.07	1.02	0.21	1.30
$\Lambda^{40}\text{Ca}$	0.03	1.05	0.21	1.29
$\Lambda^{56}\text{Fe}$	0.01	1.12	0.21	1.35
$\Lambda^{89}\text{Y}$	$6 \cdot 10^{-3}$	1.16	0.22	1.38
$\Lambda^{139}\text{La}$	$6 \cdot 10^{-3}$	1.14	0.18	1.33
$\Lambda^{208}\text{Pb}$	$1 \cdot 10^{-4}$	1.21	0.19	1.40

phase space allowed for the mesonic channel, and to smaller overlaps between the Λ wave function ψ_Λ and the nuclear surface, as A increases. The 2-body induced decay is rather independent of the hypernuclear dimension and it is about 15% of the total width. Previous works [15,16] gave more emphasis to this new channel, without, however, reproducing the experimental results. The total width is also nearly constant with A , as we already know from the experiment. In fig. 4 we compare the results from table III with recent (after 1990) experimental data for non-mesonic decay [26,27,30,32–34]. Nevertheless, we remind that the data for nuclei from $\Lambda^{28}\text{Si}$ on refer to the total width. However, as can be seen from table III, $\Gamma_M(\Lambda^{28}\text{Si})/\Gamma_{NM}(\Lambda^{28}\text{Si}) \simeq 6 \cdot 10^{-2}$ and this ratio rapidly decreases with A . The theoretical results are in good agreement with the data (which, on the other hand, have large error bars) over the whole hypernuclear mass range explored. Moreover, we also see how the saturation of the $\Lambda N \rightarrow NN$ interaction in nuclei is well reproduced.

One of the open problems in the study of weak hypernuclear decays is to understand the large experimental value of the ratio Γ_n/Γ_p which most of the present theories fail to reproduce. Only the quark model of ref. [12] predicts an enhanced ratio, although it cannot describe both mesonic and non-mesonic decays from the same basic quark hamiltonian. However, we have to remind that the data for Γ_n/Γ_p have a large uncertainty and they have been analyzed without taking into account the 3-body decay mechanism. The study of ref. [16] showed that, even if the three body reaction is only about 15 % of the total decay rate, this mechanism influences the analysis of the data determining the ratio Γ_n/Γ_p . The energy spectra of neutrons and protons from the non-mesonic decay mechanisms were calculated in ref. [17]. The momentum distributions of the primary nucleons were determined from the Propagator Method and a subsequent Monte Carlo simulation was used to account for the final state interactions. It was shown that the shape of the proton spectrum was sensitive to the ratio Γ_n/Γ_p . In fact, the protons from the three-nucleon mechanism appeared mainly at low energies, while those from the two-body process peaked around 75 MeV. Since the experimental spectra show a fair amount of protons in the low energy region they would favour a relatively larger three-body decay rate or, conversely, a reduced number of protons from the two-body process. Consequently, the experimental spectra are compatible with

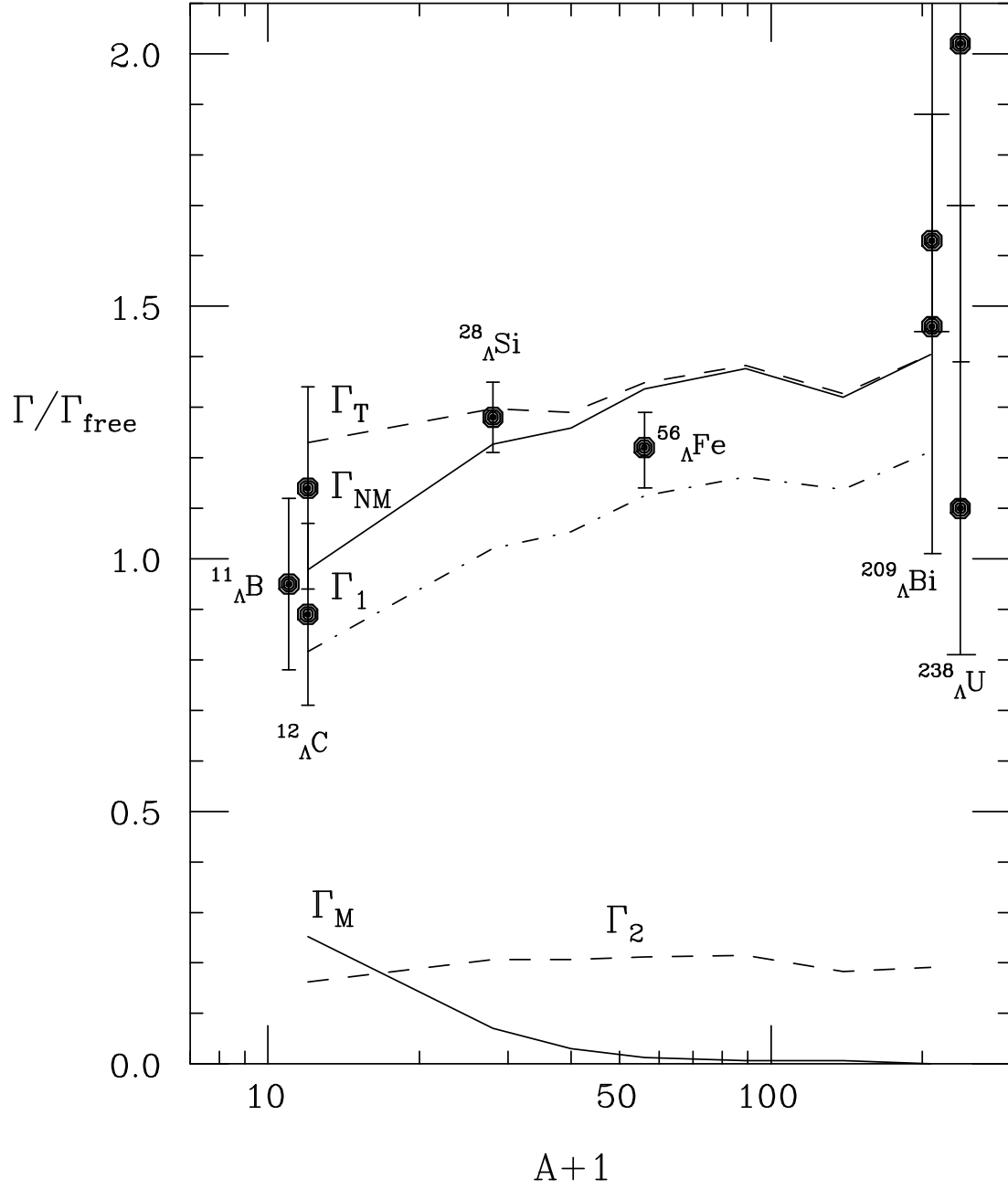


FIG. 4. Λ decay widths in finite nuclei as a function of the mass number A .

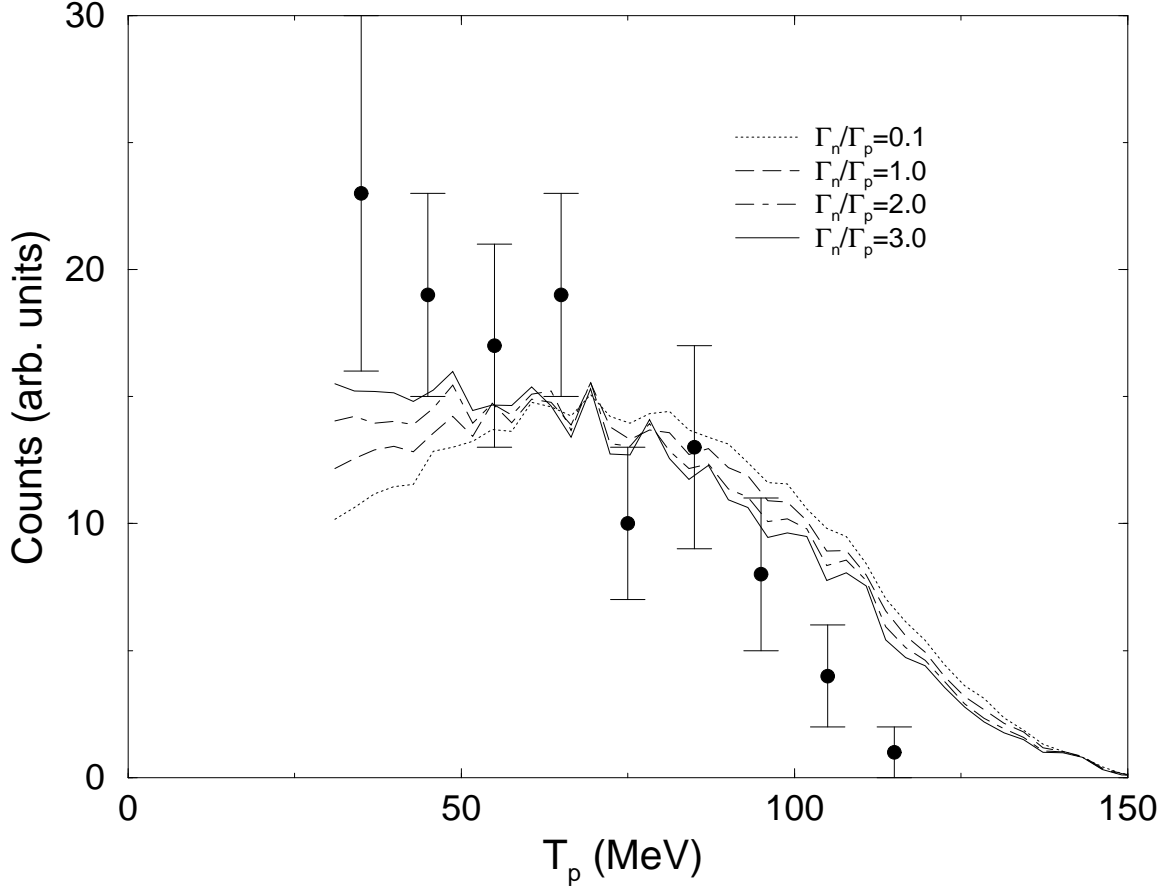


FIG. 5. Proton spectrum from the decay of $^{12}_{\Lambda}\text{C}$ for various values of Γ_n/Γ_p . The experimental data are taken from ref. [26]

values for Γ_n/Γ_p around $2 \div 3$, in strong contradiction with the present theories.

The excellent agreement with the experimental decay rates of medium to heavy hypernuclei obtained here from the Propagator Method with modified parameters, makes it worth to explore the predictions for the nucleon spectra. The question is whether this modified model affects the momentum distribution of the primary emitted nucleons strongly enough, such that good agreement with the experimental proton spectra is obtained without the need for very large values for Γ_n/Γ_p . We have thus generated the nucleon spectra from the decay of several hypernuclei using the Monte Carlo simulation of ref. [17] but with our modified g' , g'_{Λ} parameters and our more realistic nuclear density and Λ wave functions. The spectra obtained for various values of Γ_n/Γ_p , used as a free parameter in the approach of ref. [17], are compared with the BNL experimental data [26] in fig. 5. We observe that, although the non-mesonic widths are smaller by about 35% than those of refs. [16,17], the resulting nucleon spectra, once they are normalized to the same total width, are practically identical. The reason is that the ratio Γ_2/Γ_1 of two-body induced versus one-body induced

decay rates is essentially the same in both models (between 0.2 and 0.15 from medium to heavy hypernuclei), and the momentum distributions for the primary emitted protons are also very similar. As a consequence, the conclusions drawn in ref. [17] still hold and the present calculation would also favour very large values of Γ_n/Γ_p .

Therefore, the origin of the discrepancy between theory and experiment for the ratio Γ_n/Γ_p still needs to be resolved. From the theoretical side, there is still room for improving on the numerical simulation of final state interactions. In particular, Coulomb distortions and the evaporating processes need to be incorporated. We think that the evaporating process is an important ingredient which increases the nucleon spectra at low energies. Maybe this contribution is so important that there is no need for high Γ_n/Γ_p values. On the experimental side, although new spectra are now available [27,30], they have not been corrected for energy losses inside the target or detector, so a direct comparison with the theoretical predictions is not yet possible. Attempts to incorporate these corrections by combining a theoretical model for the nucleon rescattering in the nucleus with a simulation of the energy losses in the experimental set-up are now being pursued [35]. These efforts call for newer improved theoretical models that incorporate those final state interaction effects missing in ref. [17]. On the other hand, a forward step towards a clean extraction of the ratio Γ_n/Γ_p would be obtained if the nucleons from the different non-mesonic processes, $\Lambda N \rightarrow NN$ and $\Lambda NN \rightarrow NNN$ were disentangled. Through the measurement of the coincident spectra of the outgoing nucleons, it could be possible, in the near future, to split the non-mesonic decay width into its two components Γ_1 and Γ_2 [36] and obtain a cleaner measurement of the ratio Γ_n/Γ_p .

IV. CONCLUSIONS

Using the Propagator Method in Local Density Approximation, in this paper we made a new evaluation of the Λ decay widths in nuclei. Special attention has been devoted to the study of the NN and ΛN short range interactions and realistic nuclear densities and Λ wave functions were used. We have adjusted the parameters that control the short range correlations to reproduce the experimental decay widths of $^{12}_\Lambda\text{C}$. Then, the calculation has been extended to heavier hypernuclei, up to $^{208}_\Lambda\text{Pb}$. We reproduce for the first time the experimental non-mesonic decay widths from medium to heavy Λ -hypernuclei and the saturation of the $\Lambda N \rightarrow NN$ interaction is observed.

The energetic spectra of emitted nucleons calculated using the Propagator Method with modified parameters (describing the energy distributions of primary nucleons) and the Monte Carlo simulation (accounting for the final state interactions) does not change appreciably with respect to those calculated in ref. [17]. The reason is that, in spite of the fact that the non-mesonic decay widths Γ_1 and Γ_2 are sizably reduced (by about 35%) with respect to those of ref. [17], the ratio Γ_1/Γ_2 is not altered, and the momentum distributions of primary nucleons are very similar to the previous calculation. So, the conclusion drawn in ref. [17]

still holds: a comparison of the calculated spectra with the experimental one favours Γ_n/Γ_p ratios around $2 \div 3$ (or higher), in disagreement with the OPE predictions. On the other hand, we have to recall that for a clean experimental extraction of the Γ_n/Γ_p ratio it is very important to identify the nucleons which come out from the different non-mesonic processes [37].

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APPENDIX: SPIN-ISOSPIN NN AND $\Lambda N \rightarrow NN$ INTERACTIONS

In this appendix we show how the repulsive NN and ΛN Short Range Correlations (SRC) are implemented in the $NN \rightarrow NN$ and $\Lambda N \rightarrow NN$ interactions. The process $NN \rightarrow NN$ can be described through an effective potentials given by:

$$G(r) = g(r)V(r). \quad (\text{A1})$$

Here $g(r)$ is a 2-body correlation function, which vanishes as $r \rightarrow 0$ and goes to 1 as $r \rightarrow \infty$, while $V(r)$ is a meson exchange potential which in our case contains π and ρ exchange: $V = V_\pi + V_\rho$. A practical form for $g(r)$ is [25]:

$$g(r) = 1 - j_0(q_c r), \quad (\text{A2})$$

With $q_c \simeq 780$ MeV one get a good reproduction of realistic NN correlation functions obtained from G -matrix calculations. The inverse of q_c is indicative of the hard core radius of the interaction. Since there are not experimental indications, the same correlation momentum we use for the ΛN interaction. On the other hand, we remind that q_c is not necessarily the same in the two cases, given the different nature of the repulsive forces. Using the correlation function (A2) it is easy to get the effective interaction, eq. (A1), in momentum space. It reads:

$$G_{NN}(q) = V_\pi(q) + V_\rho(q) + \frac{f_\pi^2}{m_\pi^2} \{g_L(q)\hat{q}_i\hat{q}_j + g_T(q)(\delta_{ij} - \hat{q}_i\hat{q}_j)\} \sigma_i\sigma_j \boldsymbol{\tau} \cdot \boldsymbol{\tau}, \quad (\text{A3})$$

where the SRC are embodied in the correlation functions g_L and g_T . The spin-isospin $NN \rightarrow NN$ interaction can be separated into a spin-longitudinal and a spin-transverse parts, as follows:

$$G_{NN}(q) = \{V_L(q)\hat{q}_i\hat{q}_j + V_T(q)(\delta_{ij} - \hat{q}_i\hat{q}_j)\} \sigma_i\sigma_j \boldsymbol{\tau} \cdot \boldsymbol{\tau} \quad (\hat{q}_i = q_i / |\mathbf{q}|), \quad (\text{A4})$$

where

$$V_L(q) = \frac{f_\pi^2}{m_\pi^2} \left\{ \mathbf{q}^2 F_\pi^2(q) G_\pi^0(q) + g_L(q) \right\}, \quad (\text{A5})$$

$$V_T(q) = \frac{f_\pi^2}{m_\pi^2} \left\{ \mathbf{q}^2 C_\rho F_\rho^2(q) G_\rho^0(q) + g_T(q) \right\}. \quad (\text{A6})$$

In the above F_ρ is the ρNN form factor (eq. (2.6) with cut-off $\Lambda_\rho = 2.5$ GeV), and $G_\rho^0 = 1/(q_0^2 - \mathbf{q}^2 - m_\rho^2)$ is the ρ free propagator.

The $\Lambda N \rightarrow NN$ effective interactions splits into a p -wave (again longitudinal and transverse) part:

$$G_{\Lambda N \rightarrow NN}(q) = \left\{ \tilde{P}_L(q) \hat{q}_i \hat{q}_j + \tilde{P}_T(q) (\delta_{ij} - \hat{q}_i \hat{q}_j) \right\} \sigma_i \sigma_j \boldsymbol{\tau} \cdot \boldsymbol{\tau}, \quad (\text{A7})$$

with:

$$\tilde{P}_L(q) = \frac{f_\pi}{m_\pi} \frac{P}{m_\pi} \left\{ \mathbf{q}^2 F_\pi^2(q) G_\pi^0(q) + g_L^\Lambda(q) \right\}, \quad (\text{A8})$$

$$\tilde{P}_T(q) = \frac{f_\pi}{m_\pi} \frac{P}{m_\pi} g_T^\Lambda(q), \quad (\text{A9})$$

and an s -wave part:

$$\tilde{S}(q) = \frac{f_\pi}{m_\pi} S \left\{ F_\pi^2(q) G_\pi^0(q) - \tilde{F}_\pi^2(q) \tilde{G}_\pi^0(q) \right\} |\mathbf{q}|. \quad (\text{A10})$$

Form factors and propagators with a tilde imply that they are calculated changing $\mathbf{q}^2 \rightarrow \mathbf{q}^2 + q_c^2$. C_ρ is given by the expression:

$$C_\rho = \frac{f_\rho^2}{m_\rho^2} \left[\frac{f_\pi^2}{m_\pi^2} \right]^{-1}. \quad (\text{A11})$$

The expressions for the correlation functions are the following:

$$g_L(q) = - \left\{ \mathbf{q}^2 + \frac{1}{3} q_c^2 \right\} \tilde{F}_\pi^2(q) \tilde{G}_\pi^0(q) - \frac{2}{3} q_c^2 C_\rho \tilde{F}_\rho^2(q) \tilde{G}_\rho^0(q), \quad (\text{A12})$$

$$g_T(q) = - \frac{1}{3} q_c^2 \tilde{F}_\pi^2(q) \tilde{G}_\pi^0(q) - \left\{ \mathbf{q}^2 + \frac{2}{3} q_c^2 \right\} C_\rho \tilde{F}_\rho^2(q) \tilde{G}_\rho^0(q), \quad (\text{A13})$$

$$g_L^\Lambda(q) = - \left\{ \mathbf{q}^2 + \frac{1}{3} q_c^2 \right\} \tilde{F}_\pi^2(q) \tilde{G}_\pi^0(q), \quad (\text{A14})$$

$$g_T^\Lambda(q) = -\frac{1}{3}q_c^2 \tilde{F}_\pi^2(q) \tilde{G}_\pi^0(q). \quad (\text{A15})$$

Using the set of parameters:

$$q_c = 780 \text{ MeV}, \quad \Lambda_\pi = 1.2 \text{ GeV}, \quad \Lambda_\rho = 2.5 \text{ GeV}, \quad f_\pi^2/4\pi = 0.08, \quad C_\rho = 2, \quad (\text{A16})$$

at zero energy and momentum we have:

$$g_L(0) = g_T(0) = 0.615, \quad g_L^\Lambda(0) = g_T^\Lambda(0) = 0.155. \quad (\text{A17})$$

However we wish to keep the zero energy and momentum limit of $g_{L,T}$ and $g_{L,T}^\Lambda$ as free parameters, thus we replace the previous functions by:

$$g_{L,T}(q) \rightarrow g' \frac{g_{L,T}(q)}{g_{L,T}(0)}, \quad g_{L,T}^\Lambda(q) \rightarrow g'_\Lambda \frac{g_{L,T}^\Lambda(q)}{g_{L,T}^\Lambda(0)}. \quad (\text{A18})$$

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